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Where Did the Mysteries of Quantum Mechanics Arise?

— Complex Numbers, Hilbert Space, Schrödinger's Cat, Quantum Entanglement, and the Question of Physical Mechanism from the Perspective of NNRT —

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Abstract

Quantum mechanics came to employ complex probability amplitudes and the abstract state space known as Hilbert space because the behavior of electrons and light could not be described by the intuition of classical three-dimensional space alone. Complex numbers retain the magnitude and phase of an amplitude within a single quantity, while Hilbert space unifies linear superposition, inner products, observables, and time evolution in one mathematical framework. This made it possible to describe point-like particle detections and wave-like interference within the same theory. Yet this mathematical success also raised an ontological question: should the state structure in Hilbert space be read as physical reality itself? That question developed into long-standing debates over Schrödinger's cat, the measurement problem, quantum entanglement, and nonlocality. This paper traces that history for a general readership and applies to quantum theory the distinction, emphasized by NNRT in relativity, between "successful mathematics" and "the physical reality of the mathematical object." It then argues that physics remains incomplete at that stage: the next central task is to identify the physical mechanism—what acts on what, and how—that gives rise to the observed phenomena.

Central thesis: *Three questions must be distinguished: whether mathematics correctly describes a phenomenon, whether the mathematical structure itself is physical reality, and whether the physical mechanism producing the phenomenon is actually understood.*

Keywords: *Nakaza New Relativity Theory (NNRT); quantum foundations; Hilbert space; quantum entanglement; measurement problem; mathematical representation and physical reality*

1. Why Did Physics Move Toward Abstract Mathematics?

In classical mechanics through the nineteenth century, it was sufficient to regard an object as occupying some position in three-dimensional space and moving with time. Given its position and momentum, its subsequent motion could in principle be calculated.

At the beginning of the twentieth century, however, investigations of atoms, electrons, and light revealed both particle-like and wave-like properties. An electron is detected at a point on a screen, yet repeated detections behind a double slit form an interference pattern. Each individual detection is particle-like, whereas the distribution is wave-like. This dual aspect could no longer be represented adequately by the classical picture of "one particle following one trajectory." [5, 8]

Quantum mechanics therefore introduced a mathematical state space distinct from ordinary three-dimensional physical space. The most important example is Hilbert space. The essential point is that Hilbert space was not introduced as "another physically existing universe," but as a mathematical framework in which quantum states can be added, inner products can be taken, probability amplitudes can be calculated, and observables can be represented as operators. [7, 8]

2. What Hilbert Space Made Possible

In Hilbert space, a quantum state is represented as a vector. If two states exist, their linear combination can also be treated as a state. [7, 8]

$$\psi = \alpha A + \beta B$$

There is nothing strange about this expression within Hilbert space. In a vector space, forming the sum of two vectors is a basic operation. Superposition therefore appears as a mathematically natural structure.

When position is measured, the state vector can be expanded in the position basis, and the probability can be calculated from the resulting amplitude. [6–8]

$$P(x) = \psi\_R^2(x) + \psi\_I^2(x)$$

In the double-slit experiment, the amplitudes associated with the two paths are added before the absolute square is taken, and an interference term therefore appears. [5, 6]

$$\psi(x) = \psi_1(x) + \psi_2(x)$$

$$P(x) = P_1(x) + P_2(x) + I_{12}(x)$$

This mathematics reproduces experimental results with extraordinary success. This is one of the great achievements of quantum mechanics. [5, 6, 8]

3. Why Complex Numbers? — Combining Magnitude and Phase in One Quantity

An important feature of quantum mechanics is that Hilbert space is not merely a "box containing many states." In standard quantum mechanics, the state space is a Hilbert space over the complex numbers, and the coefficients of a state are themselves treated as complex probability amplitudes. [6–8]

$$z = r\, exp(i\theta)$$

A complex number z can retain both an absolute magnitude r and a phase θ. The squared magnitude is related to probability, while phase differences become observable through interference. Complex numbers are therefore useful in quantum theory not simply because they place a real and an imaginary component side by side, but because they represent amplitude magnitude and phase rotation within one quantity. [6, 8]

$$z * z = r^2$$

In the double-slit experiment, what is associated with each slit is not a probability itself but a complex amplitude. The amplitudes are added first, and only afterward is the absolute square taken. This is why an interference term appears that would not arise from simply adding ordinary probabilities. [6, 8]

$$P = (\psi_1 + \psi_2) * (\psi_1 + \psi_2)$$

It is important not to interpret the real part of a complex number as "the particle" and the imaginary part as "the wave." Particle and wave properties are not packed into the two components of a complex number. Rather, complex amplitudes and linear state space provide the mathematical framework in which wave-like phase interference is retained while measurements yield particle-like discrete records. [6–8]

Special states or representations can sometimes be expressed using only real numbers, but the complex structure plays an essential role in naturally preserving general time evolution, relative phase, and interference in standard quantum mechanics. The imaginary unit i in the Schrödinger equation is deeply connected with time evolution that rotates phase while preserving the norm of the state. [5, 7, 8]

$$i\hbar\, \partial\psi/\partial t = \hat{H}\psi$$

4. Hilbert Space Is Not a "Spacetime That Mixes Particles and Waves"

A terminological distinction is important here. Hilbert space is not "Hilbert spacetime." It is not a spacetime that represents physically existing time and space; it is a state space used to represent quantum states. [7, 8]

$$\text{physical space} \neq \text{Hilbert space}$$

Hilbert space allows different states to be added as vectors, relations among states to be measured through inner products, observables to be represented by operators, and time evolution to be described without losing the phase information of complex amplitudes. Its role is therefore best described not as "a space in which particles and waves are directly added," but as "a state space that preserves quantum superposition and phase and thereby describes particle-like detection and wave-like interference within one theory." [7, 8]

This distinction may appear small, but it is crucial. If Hilbert space is regarded as an extension of physical space, one is tempted to interpret coexisting state components as simultaneous existence in physical spacetime. Yet the coexistence of several components within a mathematical state space and the simultaneous physical existence of one object in several states within real three-dimensional space are different propositions from the outset.

5. A New Question Appeared at the Moment the Mathematics Succeeded

The success of the mathematics and the claim that the mathematical world itself is physical reality are not the same. This is where the ontological problem of quantum theory begins.

"The sum of states gives the correct experimental result" and "the physical object actually exists simultaneously in two states" are logically distinct claims.

In the double-slit experiment, states corresponding to two paths are added in Hilbert space, and the interference pattern is thereby calculated correctly. This naturally invites the question: "Did the electron itself really pass through both slits at the same time?"

Yet that question already contains an assumption: it tries to read a state representation in Hilbert space directly as the physical mode of existence of an electron in real three-dimensional space. Mathematics may predict the observations perfectly without automatically telling us what the mathematical state itself physically is.

6. "Map" and "Territory" — A First Analogy

The distinction resembles the relation between a map and the territory. A map can display several possible roads from one point to a destination at the same time. There is no contradiction in drawing them together on one sheet.

But the fact that two roads are drawn simultaneously on a map does not mean that one car is physically traveling along both roads at the same time. The map may correspond closely to reality, but the map itself is not the territory.

Of course, a quantum state is richer than an ordinary road map. Its components possess phases and can interfere with one another. It is therefore also inadequate to regard the quantum state as merely a list of our ignorance. Even so, "representation" and "reality" must be distinguished.

7. 1935 — The Cat and Entanglement Emerged from the Same Problem

In 1935, Einstein, Podolsky, and Rosen published the EPR paper, asking whether the quantum-mechanical state description gives a complete account of physical reality. In the same year, Schrödinger explicitly discussed entanglement and introduced his famous cat thought experiment. [9, 10]

These are not two unrelated curiosities. Both arose from the same question: to what extent should the mathematics of the quantum state be read as physical reality itself? [9, 10]

8. Schrödinger's Cat — Why Does It Look So Strange?

If the decay of a radioactive atom is mechanically linked to whether a cat lives or dies, the linear time evolution of quantum mechanics allows the composite system to be written as a superposition. [10]

$$\Psi = \alpha(decay\ /\ dead) + \beta(no\ decay\ /\ alive)$$

This leads to the familiar question: "Until the box is opened, is the cat both alive and dead?" What must first be recognized, however, is that the equation represents the state of the composite system in Hilbert space. [10]

Within Hilbert space, it is mathematically natural for both components to coexist as one state. The paradox appears when that structure is mapped directly onto the cat in physical space and interpreted as meaning that one real cat literally exists both alive and dead.

The core of the cat problem is therefore not whether superposition is required in the calculation, but whether the mathematical state structure called superposition should be identified with the physical mode of existence of the real cat.

9. Quantum Entanglement — Do Not Confuse Mathematical Nonseparability with Physical Space [9, 12]

For two particles A and B, there are cases in which the total state cannot be factorized into a product of individual states. This is quantum entanglement.

$$\boldsymbol{\mathcal{H}}_{AB} = \boldsymbol{\mathcal{H}}_{A} \otimes \boldsymbol{\mathcal{H}}_{B}$$

$$\Psi_{AB} \neq \psi_A \otimes \psi_B$$

What this equation says is that the composite state is nonseparable in Hilbert space. When A and B are separated far apart in physical space, however, another question arises: "If they still form one state, must something also connect them instantaneously in physical space?"

Here again, state space must be distinguished from physical space. Nonseparability in Hilbert space does not by itself imply the existence of an invisible physical thread in three-dimensional space. Bell-type experiments strongly constrain simple locally predetermined models, but they do not establish an identity between Hilbert space and real three-dimensional space. [12, 13]

10. The Measurement Problem — Why Does Only One Result Appear?

A quantum state evolves continuously according to the Schrödinger equation, whereas an experiment records one definite result on each run. The tension between these two descriptions lies at the center of the measurement problem. [7, 8, 14]

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{\mathrm{H}}\psi$$

From the state vector, the Born probability for each possible result can be calculated. [6–8]

$$P_i = A\_Ri^2 + A\_Ii^2$$

But the ability to calculate probabilities is not the same as understanding the physical mechanism by which, in this particular trial, result i rather than another result was realized. This remains one of the fundamental issues behind the continuing divergence of interpretations. [7, 8, 14]

11. The Same Category Error Appeared in Relativity — The Starting Point of NNRT [1–4]

At this point the way NNRT frames the problem of relativity becomes important. The mathematics of relativity has described redshift, light deflection, Shapiro delay, perihelion advance, GPS, gravitational-wave observations, and many other phenomena with high precision. NNRT does not deny this success. [1–4]

NNRT raises a question at a different level. Does the success of the mathematics of relativistic measurement spacetime, constructed through electromagnetic waves and clocks, require the conclusion that this four-dimensional curved spacetime is itself the real spacetime? [1, 3, 4]

The fact that mathematics reproduces observational results perfectly is not identical to the claim that the corresponding mathematical geometry is itself physical reality.

NNRT retains universal time and universal three-dimensional space as the real physical basis, and interprets relativistic measurement spacetime as a structure constructed through electromagnetic phase, frequency, wave number, and related observables. [2–4]

$$\boldsymbol{\mathcal{M}}_{\mathrm{U}} \rightarrow K_\mu \rightarrow (\omega, k, \Theta) \rightarrow g_{\mu\nu}^{\mathrm{obs}}$$

The essential point is not to discard the mathematics of curved spacetime, but to reinterpret what that successful mathematics is describing. [3, 4]

12. What Becomes Visible When the Same Distinction Is Applied to Quantum Theory? [4]

The same distinction can be applied to quantum theory.

$$\mathbb{R}^3 \neq \boldsymbol{\mathcal{H}}$$

Real three-dimensional space and Hilbert space are different mathematical objects from the outset. Therefore, superposition and nonseparability in Hilbert space need not immediately be interpreted as double existence or geometric nonlocality of real three-dimensional space itself. [4, 7, 8]

This distinction suggests that part of the strangeness of the cat and entanglement may arise from identifying mathematical representation with physical reality.

13. But We Must Not Say "The Problem Is Solved" at This Point

A crucial distinction now appears. Separating mathematics from reality can clarify an ontological confusion, but physics is not complete merely because that confusion has been removed.

"Do not identify representation with reality" is a first resolution. The next question is: what, on the side of reality, acts on what, and how?

The same applies to NNRT. It has progressed to the point of interpreting gravitational redshift not as "the stretching of real spacetime itself," but as a gravitational action that appears in the phase, frequency, and wave number of electromagnetic radiation and is then reconstructed as measurement spacetime. [4]

Yet a further question remains: What is gravity physically? What acts on the electromagnetic field? Which degrees of freedom couple to which, and why does that coupling produce the observed phase shift?

$$\frac{dK_\mu}{d\lambda} = -\frac{1}{2}\,\partial_\mu C^{\alpha\beta}\,K_\alpha\,K_\beta$$

The phase-transport equation specifies mathematically how phase changes. But by itself it is not yet the final microscopic physical explanation of why that interaction exists. [4]

14. Three Levels — Description, Ontology, and Mechanism

The location of the problem becomes clearer if it is organized into three levels.

Level	Question	Example
Level 1: Mathematical description	Which equations correctly reproduce observations?	Hilbert space, Born rule, metric, phase transport
Level 2: Ontology	What should the mathematical object be taken to represent?	Quantum state ≠ reality itself; measurement spacetime ≠ real spacetime
Level 3: Physical mechanism	What acts on what, and how?	Mass, gravity, quantum fields, and the physical basis of interaction

Modern physics achieved extraordinary success at Level 1. At Level 2, long debates arose over how the mathematics should be interpreted. NNRT retains the successful mathematics of relativity while attempting to dissolve an identification made at Level 2. When the same perspective is applied to quantum theory, the next task at Level 3 becomes explicit. [1–4]

15. Does Physics End with the Claim that "God Is a Mathematician"?

Because mathematical laws describe the world with astonishing accuracy, mathematical physics naturally invites the thought that the essence of nature may itself be mathematics. The attraction of that idea is understandable. Complex numbers, Hilbert space, and four-dimensional metrics are not merely convenient symbols; they capture the regularities of phenomena with remarkable depth. Yet the statement that a mathematical structure represents nature accurately is different from the statement that the mathematical object itself is physical reality.

Likewise, "the mathematics predicts perfectly" is not the same as "we physically understand why the phenomenon occurs." Newton's equation of motion can predict acceleration correctly while leaving open the further question of why mass resists acceleration. A gravitational-redshift formula can be exact while the physical question still remains: what acts on the electromagnetic wave and changes its phase?

The point is therefore not to doubt mathematics. It is to recognize the success of mathematics fully while refusing to treat that success itself as the final answer to ontology.

Even if a map represented a city perfectly and predicted traffic flow exactly, the lines on the map would not physically move the automobiles.

16. NNRT's Next Central Question I — What Is Mass?

In NNRT, the relation connecting mass with an intrinsic phase scale occupies an important position.

$$\hbar\omega_0 = mc^2$$

This relation connects mass with an intrinsic frequency and phase scale. But it does not by itself mean that the physical reality of mass has been explained. The next question is why such an internal phase structure appears physically as inertia—resistance to acceleration.

intrinsic phase structure → inertia → resistance to acceleration

The old question "What is the hard, heavy reality we call mass?" should therefore not be regarded as closed merely because a relation to phase has been found. Rather, that relation provides an entrance to the deeper problem of a concrete field-interaction mechanism.

17. NNRT's Next Central Question II — What Is Gravity?

NNRT does not begin by identifying gravity with the geometric deformation of real spacetime itself. It instead interprets the gravitational action generated by matter sources as acting on field propagation, especially phase structure, and thereby changing observables.

$$\text{source} \rightarrow \text{gravitational action} \rightarrow \text{phase response} \rightarrow \text{observables} \rightarrow \text{measurement spacetime}$$

But the "gravitational action" placed at the beginning of this chain must itself be examined physically. Which property of the source generates the field? With which variables of the electromagnetic field does it couple? Why does that coupling appear as redshift, deflection, and delay? This is the gravitational problem at the third level.

18. NNRT's Next Central Question III — What Is the Real Structure of the Quantum Field?

The same issue arises on the quantum side. It is not enough merely to separate the state in Hilbert space from physical reality. One must then ask what exists on the side of the real quantum system.

$$\psi \neq \text{physical reality itself}$$

The quantum state is not merely subjective information. It is a physically meaningful state representation that accurately generates interference and correlations. Yet the relation between the underlying real field structure and its representation in Hilbert space remains to be clarified. [14]

From this viewpoint, the measurement problem is transformed. Instead of asking "Why does the wave function collapse as if by magic?", one asks: "Which physical process involving the real field, the apparatus, and their interaction generates one definite observational record?"

19. The Unresolved Problems of Relativity and Quantum Theory Converge on One Question

In relativity, the metric is not identified with real spacetime itself. In quantum theory, the state vector is not identified with reality itself. What then remains is a strikingly similar question in both cases.

$$\text{What is the physical structure of reality behind the successful mathematical representation?}$$

NNRT can therefore be understood not as an attempt to destroy successful mathematics, but as an attempt to relocate that mathematics to the appropriate descriptive layer so that the physical mechanism behind it can once again be asked. [1–4]

20. Stage, Score, and Recording — A Final Analogy

The three levels can be understood intuitively by comparing the world to a concert.

Analogy	Correspondence in physics
Stage	The physically real world
Score	Mathematical state representation / theory
Interaction between performer and instrument	Physical interaction mechanism
Recording	Observational record

If a musical score is sufficiently accurate, it can predict which sounds will be heard. But the score itself does not physically produce the sound. In reality, a performer acts on an instrument, the air vibrates, and a microphone records the result.

Physics is similar. Mathematics can be an extraordinarily accurate score. Yet what physics ultimately seeks to know is not only whether the score is correct, but what acts on what on the stage and thereby produces the sound.

21. What the Whole Story Reveals

Quantum mechanics acquired the powerful mathematical arena of Hilbert space because classical intuition was insufficient to describe the observed phenomena. That mathematics described the double slit, atoms, light, and entanglement with remarkable accuracy. Precisely because its success was so great, the question arose of how the mathematical state should be understood as physical reality, leading to the long debates over the cat, measurement, and nonlocality. [5–14]

Relativity likewise explained many observations through the extraordinarily powerful mathematics of the four-dimensional metric. An ontology then arose in which that mathematical geometry was read as real spacetime itself. NNRT separates mathematical success from that ontological identification and relocates four-dimensional relativistic spacetime as measurement spacetime constructed through observation, including electromagnetic measurement. [1–4]

Applying the same perspective to quantum theory allows superposition and nonseparability in Hilbert space to be distinguished from double existence or geometric nonlocality in physical space itself. Part of the "strangeness" of quantum mechanics can then be reconsidered as a problem created by identifying representation with reality. [4, 7–14]

But the story does not end there. Once mathematics and reality are separated, genuinely difficult questions remain. What is mass? What is gravity? What is the real structure of the quantum field? What acts on electromagnetic radiation, what produces inertia, and what produces one definite detection record?

22. Conclusion — From Mathematics Telling Us "What Happens" to Physics Asking "Why It Happens"

The conclusion of this paper is not that mathematics should be valued less. On the contrary, it is that the success of mathematics should be fully recognized while the meaning of that success is properly delimited.

Mathematics can tell us with extraordinary precision what will happen. But what physics ultimately seeks to know is what acts on what, how it acts, and why the phenomenon occurs.

NNRT has advanced, in relativity, to the point of separating "how things appear in measurement" from "what is physically real." In quantum theory, separating the state in Hilbert space from the real world likewise suggests a new way of organizing the problems of the cat and entanglement. [1–4]

The next stage is not merely another interpretation. It is the construction of a concrete physical mechanism at the level of reality: why the phase structure associated with mass produces inertia; what the gravitational action physically is and how it couples to the electromagnetic field; and how the real structure of a quantum field, through interaction with an apparatus, generates one definite record.

This is the third stage toward which NNRT should proceed. The task is not to doubt mathematics, but to ask again, as physics, what is actually occurring in the real world behind the remarkable regularities revealed by mathematics. [4]

Therefore, the metaphor that "God was a mathematician" cannot yet be asserted as the final ontology of nature, at least at the present stage of physics. The fact that mathematics describes the world with astonishing accuracy is not identical to the proposition that the world itself is exhausted by mathematical objects. Physics still has work to do: it must ask what acts on what, and how, behind the mathematics.

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